

Initial Conditions for Imperfect Dark Matter

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Abstract

We discuss initial conditions for the recently proposed Imperfect Dark Matter (Modified Dust). We show that they are adiabatic under fairly moderate assumptions about the cosmological evolution of the Universe at the relevant times.

1 Introduction and Summary

Planck data favors adiabatic initial conditions at the onset of the hot Big Bang [1]. This corresponds to the situation in the early Universe, when the number densities of different particle species, e.g., dark matter (DM) particles and photons, are universally distributed in space. That picture, comfortably accommodated in the Λ CDM cosmology supplemented by the short stage of an inflationary expansion, can be less transparent in more exotic setups. In particular, adiabaticity of initial conditions is obscured in the case, if DM has a non-particle origin.

In the present paper, we continue to discuss Imperfect Dark Matter (IDM) scenario [2, 3] (Modified Dust in Ref. [2]). The action of IDM is given by

$$S = \int d^4x \sqrt{-g} \left[\frac{\lambda}{2} (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 1) + \frac{\gamma(\varphi)}{2} (\Box \varphi)^2 \right], \quad (1)$$

(see Ref. [4] for generalizations). Here λ and φ are two scalars, and $\gamma(\varphi)$ is some function of the field φ . The Lagrange multiplier λ enforces the constraint $g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi = 1$. This equation defines the geodesics followed by the collisionless particles, with the field φ being the velocity potential. The analogy with the pressureless perfect fluid, dust, is complete in

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the situation when the higher derivative (HD) term is absent, i.e., $\gamma(\varphi) = 0$. In that case, the Lagrange multiplier λ plays the role of the energy density of dust: it redshifts away as the inverse third power of the scale factor a in the expanding Universe. While the dust provides a good description of DM to the linear level, it should be modified in the non-linear regime. The reason is that it develops caustic singularities, i.e., the physical quantities,—the velocity dispersion and the energy density,—blow up at the finite time [5, 6]. This drawback of the dust model was one of the motivations (not the unique one, though) to introduce the HD term [2]. Before we summarize some of the effects arising due to the non-zero function $\gamma(\varphi)$, let us discuss, how the action (1) emerges in different physical frameworks.

The mimetic dark matter scenario [7, 8] deals with the conformally transformed metric of the form¹,

$$g_{\mu\nu} = (\tilde{g}^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi) \tilde{g}_{\mu\nu} .$$

Rather unexpectedly, variation of the general relativity action with respect to the auxiliary metric $\tilde{g}_{\alpha\beta}$ and the scalar φ , gives more than just Einstein's equations. The difference is about a new degree of freedom behaving as dust. This returns us to the action (1) up to the HD term [10], which is set by hands in this context [8]. Alternatively, one can trace back the origin of IDM to Lorentz violating theories of gravity. In particular, the action (1) arises in the infrared limit of the Horava–Lifshitz model with the projectability condition applied [11, 12, 13]. More generically, IDM is closely related to a version of Einstein–Aether theory [14] with the aether field being the derivative of the scalar [15, 16] (see also Refs. [17, 18, 19] for the most recent discussion on the topic).

During the large part of IDM evolution, one assumes that the function $\gamma(\varphi)$ is constant. In this situation, the model possesses shift symmetry $\varphi \rightarrow \varphi + c$. There is, consequently, the Noether charge density that redshifts away as the inverse third power of the scale factor. Up to the term suppressed by the γ factor, the energy density of IDM is equal to the Noether charge density [3] (Section 2). Hence, with a good accuracy, the cosmological evolution of IDM mimics that of the dust particles². The degeneracy gets broken at the linear level, where the γ -term leads to the constant sound speed $c_s^2 \simeq \gamma$ [8]. This sets a cutoff on the power spectrum at sufficiently small scales: beyond the sound horizon energy density perturbations do not grow. Therefore, they are suppressed compared to the predictions of cold dark matter (CDM) scenarios. In particular, setting³ $\gamma \sim 10^{-9}$, one can suppress the growth of structures with the comoving wavelength $\lesssim 100$ kpc, alleviating the mismatch between the observed number of dwarf galaxies and the value predicted in the CDM framework [20, 21,

¹The generalization of mimetic dark matter scenario to disformal metrics transformations has been constructed in Ref. [9].

²This is an exact statement in the situation, when IDM is the only component of the Universe. See the discussion in Section 2

³Note that we use the convention $8\pi M_{Pl}^2 = 1$ throughout the paper, where M_{Pl} is the Planck mass. An associated value of the parameter γ reads in units of Ref. [2] $\gamma \sim 10^{-10} M_{Pl}^2$.

22]. Alternatively, however, the small scale problems may be the consequence of the incorrect implementation of several baryonic processes, as indicated most recently in the analysis of Ref. [23]. In that case, one rather deals with the constraint [2]

$$\gamma \lesssim 10^{-9} . \quad (2)$$

For those small values of the parameter γ , the linear evolution of IDM perturbations is analogous to that of CDM given that they start from the same initial conditions (see the discussion below). The important difference, however, may arise at the structure formation level, i.e., in the non-linear regime [2]. This line of discussion is far out of the scope of the present paper. Here we will be interested in the opposite situation, namely, when the relevant cosmological modes are in the deep super-horizon regime.

If the shift symmetry is exact at *all* the times, IDM cannot be the main component of the invisible matter in the Universe. The reason is that the Noether charge density gets washed out during inflation with an exponential accuracy. In this situation, IDM constitutes only a tiny, $\mathcal{O}(\gamma)$, fraction of the overall DM during the dust dominated stage [3]. To avoid this, one necessarily assumes breaking of the shift symmetry taking place at the early stages of the Universe, i.e., deeply in the radiation dominated (RD) era. In IDM model, this is realized by promoting the constant γ to the function $\gamma(\varphi)$ ⁴. We assume furthermore that the variation of the function $\gamma(\varphi)$ is substantial only at very early times, and negligible otherwise. In that way, one can easily generate the amount of the Noether charge required to explain cosmological experiments, as we review in Section 2. Notably, perturbations of IDM produced by the same mechanism are adiabatic with a high accuracy.

Before the Noether charge gets produced, IDM tracks the dominant matter of the Universe, e.g., it has an equation of state of radiation in the Universe driven by the relativistic particles. At the linear level, the similarity with the dominant matter reveals in an exact adiabaticity of IDM perturbations (Section 3)⁵. That situation is quite analogous to what one has in the Λ CDM cosmology at very high temperatures, when all the particle species are in the thermodynamic equilibrium. More importantly, the adiabaticity of IDM perturbations holds later on, after the shift-symmetry breaking occurs, despite the presence of the non-adiabatic pressure. We prove this by the explicit computation of the curvature perturbation of IDM, i.e., ζ_{IDM} . We show that the latter corresponds to the adiabatic initial conditions for IDM, at least under rather moderate assumptions: the variation of the gravitational potential must be negligible relatively to the Hubble rate at the times, when the shift-symmetry

⁴Another idea would be to couple the field φ to the inflaton [7]. In the present paper, however, we consider only gravitational interactions between IDM and other fields.

⁵The analogous observation has been made in the earlier work [2]. Here we show that this is an exact statement. We also generalize it to any dominant matter in the Universe, while the case of radiation has been considered in Ref. [2].

breaking takes place. That condition is obeyed with a high accuracy well before the matter/radiation equality. In particular, this guarantees that IDM is indistinguishable from CDM at the level of cosmic microwave background measurements⁶.

The outline of the paper is as follows. In Section 2, we briefly review the IDM scenario including the mechanism for producing the Noether charge density in that picture. In Section 3, we discuss the super-horizon evolution of IDM perturbations and show that they are adiabatic under rather general conditions.

2 Generating dark matter

We start with writing down the system of equations following from the action (1). The simplest one is the constraint

$$g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi = 1 , \quad (3)$$

obtained from the variation with respect to the Lagrange multiplier. Applying the covariant derivative to the constraint (3), one obtains the geodesic equation. In this regard, IDM is equivalent to the collection of dust particles moving in the gravitational field. This degeneracy, however, gets broken at the level of the other equations. Variation of the action (1) with respect to the field φ yields [3]

$$\nabla_\mu J^\mu = \frac{1}{2}\gamma_\varphi(\Box\varphi)^2 , \quad (4)$$

where

$$J^\mu = (\lambda - \gamma_\varphi\Box\varphi)\partial^\mu\varphi - \gamma(\varphi)\partial^\mu\Box\varphi . \quad (5)$$

Finally, the energy-momentum tensor corresponding to the action (1) is given by [3, 8]

$$\begin{aligned} T_\nu^\mu = & \left(\lambda - 2\gamma_\varphi(\varphi)\Box\varphi \right) \partial_\nu\varphi\partial^\mu\varphi + \gamma(\varphi) \left(\partial_\alpha\varphi\partial^\alpha\Box\varphi + \frac{1}{2}(\Box\varphi)^2 + \frac{\gamma_\varphi(\varphi)}{\gamma(\varphi)}\Box\varphi \right) \delta_\nu^\mu - \\ & - \gamma(\varphi) (\partial_\nu\varphi\partial^\mu\Box\varphi + \partial_\nu\Box\varphi\partial^\mu\varphi) . \end{aligned}$$

In the present paper we are mostly interested in the background evolution of IDM as well as in the linear evolution of its super-horizon perturbations. That is, we will always neglect spatial derivatives of the fields. In this approximation, Eq. (4) takes the form⁷,

$$\frac{d}{dt}(\sqrt{-g}n) = \frac{1}{2}\sqrt{-g}\gamma_\varphi(\varphi)(\Box\varphi)^2 , \quad (6)$$

⁶This way of setting initial conditions is to be compared to that of Ref. [2]. In the latter paper, IDM perturbations relied on the arbitrary constant of integration. That is, the adiabaticity of initial conditions in Ref. [2] was at the price of tuning an arbitrary constant to some particular value. Alternatively, any other choice would result in an admixture of an isocurvature mode.

⁷We stick to the notion "Noether charge", that can be misleading at the times, when the shift symmetry is broken. Hopefully, this is not going to confuse the reader.

where $n \equiv J^0$ is the Noether charge density. We assume that during most of the stages of the evolution, the function $\gamma(\varphi)$ is constant, i.e., the model is shift symmetric. In that case, the r.h.s. of Eq. (4) is zero and one has

$$\frac{d(na^3)}{dt} = 0 \quad n = \lambda - 3\gamma\dot{H} . \quad (7)$$

As it follows, the Noether charge density redshifts away as $1/a^3$, i.e., $n = C/a^3$. Assuming that IDM obeys shift symmetry during the inflationary expansion of the Universe, the density n relaxes to zero with an exponential accuracy, i.e., $n = 0$ at the onset of the hot Big Bang. Otherwise, one would need to set an extremely large value of n in the beginning of inflation.

In the situation with the zeroth Noether charge, IDM tracks the total matter filling in the Universe [3]. Indeed, the physical energy density is related to the Noether charge density by

$$\rho_{IDM} \equiv T_0^0 = n + \frac{3\gamma}{2}\rho_{tot} , \quad (8)$$

while the pressure is given by

$$\mathcal{P}_{IDM} \equiv -\frac{1}{3}T_i^i = \frac{3\gamma}{2}\mathcal{P}_{tot} - 3\dot{\gamma}H . \quad (9)$$

Here we made use of equations $H^2 = \frac{1}{3}\rho_{tot}$ and $-2\dot{H} - 3H^2 = \mathcal{P}_{tot}$; the subscript 'tot' stands for the total matter including IDM. Neglecting the Noether charge density as well as the variation of the parameter γ in Eqs. (8) and (9), one obtains $w_{IDM} \equiv \frac{\mathcal{P}_{IDM}}{\rho_{IDM}} = w_{tot}$, where $w_{tot} \equiv \frac{\mathcal{P}_{tot}}{\rho_{tot}}$. This proves the point made above.

For the future convenience, we prefer to rephrase Eqs. (8) and (9) by splitting the contributions to the total energy density and pressure, which follow from IDM and the external matter fields. This gives

$$\rho_{IDM} = \frac{2n}{2-3\gamma} + \frac{3\gamma}{2-3\gamma}\rho_{ext} \quad (10)$$

and

$$\mathcal{P}_{IDM} = \frac{3\gamma}{2-3\gamma}\mathcal{P}_{ext} - 6\frac{\dot{\gamma}}{2-3\gamma}H . \quad (11)$$

Here $\rho_{ext} \equiv \rho_{tot} - \rho_{IDM}$ and $\mathcal{P}_{ext} \equiv \mathcal{P}_{tot} - \mathcal{P}_{IDM}$ denote the total energy density and pressure of all the matter fields not including IDM, respectively. As it follows from Eqs. (10) and (11), IDM behaves as a perfect tracker also with respect to the external matter. Namely, setting the Noether charge density and the derivative of the parameter γ to zero, we have $w_{IDM} = w_{ext}$, where $w_{ext} \equiv \frac{\mathcal{P}_{ext}}{\rho_{ext}}$. In particular, this means that for a very small value of the parameter γ , IDM may constitute only a tiny fraction of the invisible matter during the dust dominated stage. We are interested in the different opportunity of IDM being the main (the only) component of DM.

Since this point on and until the end of the Section, we assume that there is a period of shift-symmetry breaking taking place deeply in the RD stage. This allows to generate non-zero Noether charge, which, by the suitable choice of parameters, can be tuned to match the observed value [3]. With no much loss of generality, we consider an instantaneous transition from some initial value γ_1 to the value $\gamma_2 > \gamma_1$. Namely, the function $\gamma(\varphi)$ has the form

$$\gamma(\varphi) = \gamma_1 + \Delta\gamma\sigma(\varphi - \varphi_*) , \quad (12)$$

where $\sigma(\varphi - \varphi_*)$ is the Heaviside function; φ_* is the constant pointing the time, when the transition happens, and $\Delta\gamma \equiv \gamma_2 - \gamma_1$.

Accordingly, the value of the Noether charge undergoes an instantaneous flip. Integrating Eq. (6) with the initial condition $n = 0$ set at $t < t_*$, we obtain

$$na^3 = \frac{9}{2}\Delta\gamma \cdot H^2 a^3 \big|_{t=t_*} \quad t > t_* . \quad (13)$$

One can tune the time and the quantity $\Delta\gamma$, so that we get the correct energy density of DM today. As it follows, earlier the Noether charge is generated, less value of $\Delta\gamma$ is required. In particular, for the choice $\Delta\gamma \lesssim \gamma \lesssim 10^{-9}$, this indeed happens at very early times. The corresponding redshifts and temperatures read $z \gtrsim 10^{12} - 10^{13}$ and $T \gtrsim 100 \text{ MeV} - 1 \text{ GeV}$, respectively. Soon after the Noether charge is produced, it becomes the total contribution to the energy density of IDM. Indeed, the second term in Eq. (10) mimicking the behaviour of the external matter of the Universe (radiation) redshifts away fast relatively to the Noether charge density. Since this point on, the cosmological evolution of IDM resembles that of the dust particles. This is an exact statement in the situation, when IDM is the only matter in the Universe. In that case, $\rho_{IDM} = \rho_{tot}$ and $\mathcal{P}_{IDM} = \mathcal{P}_{tot}$. From Eqs. (8) and (9), one then easily obtains $\rho_{IDM} \propto n \propto 1/a^3$ and $\mathcal{P}_{IDM} = 0$ [2, 3, 15].

In the remainder of the paper, we show that perturbations of IDM generated by the same mechanism are adiabatic under fairly relaxed assumptions.

3 Super-horizon evolution of IDM perturbations

3.1 Generalities

Before we dig into the details of the linear level analysis, let us specify the gauge choice. Generically, the metric reads to the linear order,

$$ds^2 = (1 + 2\Phi)(dx^0)^2 + 2a\partial_i Z dx^0 dx^i - a^2(\delta_{ij} + 2\Psi\delta_{ij} - \partial_i\partial_j E)dx^i dx^j ,$$

where E , Z , Φ and Ψ are the scalar potentials, and we omitted vector and tensor perturbations. See Refs. [24, 25, 26] for the reviews and textbooks on the cosmological perturbation

theory. Calculations look particularly simple and transparent in the synchronous gauge, which we use in the bulk of the paper. In the Appendix A, we provide calculations in the Newton's gauge in order to cross-check the results.

To go from the Newton's gauge, where $Z = 0$ and $E = 0$, to the synchronous gauge, one makes the coordinate transformation, $\tilde{x}_\mu = x_\mu + \xi_\mu$, where $\xi_0 = \delta\varphi$ and ξ_i obey the condition $\partial_0\xi_i + \partial_i\xi_0 = 0$. With this coordinate choice, the perturbation $\delta\varphi$ and the potential Φ turn into zero,

$$\delta\varphi \rightarrow \delta\tilde{\varphi} = 0 \quad \Phi \rightarrow \tilde{\Phi} = 0 .$$

The fact that perturbations of the field φ and the potential Φ can be switched to zero simultaneously follows from the gauge-independent relation $\delta\dot{\varphi} = \Phi$,—the constraint (3) linearized. In the synchronous gauge, the potential Ψ is given by

$$\Psi \rightarrow \tilde{\Psi} = \Psi - H\delta\varphi . \quad (14)$$

The condition $\partial_i\xi_0 + \partial_0\xi_i = 0$ then guarantees that the $(0i)$ -component of the metric remains zero. At the same time, the potential E is generically non-zero in the synchronous gauge. This is, however, negligible in the super-horizon regime, which is the case of our primary interest. Indeed, $\partial_i\partial_j\tilde{E} = -(\partial_i\xi_j + \partial_j\xi_i)/a^2 \propto \partial_i\partial_j\delta\varphi/(a^2H) \rightarrow 0$.

Hereafter, we prefer to omit the tilde over the transformed quantities. In the synchronous gauge, the Noether charge density perturbation is given by (see Eq. (5)),

$$\delta n = \delta\lambda - 3\gamma\ddot{\Psi} - 3\dot{\gamma}\dot{\Psi} . \quad (15)$$

In what follows, we will also need the expressions for IDM energy density and pressure perturbations. These are given by

$$\delta\rho_{IDM} = \delta n + 9\gamma H\dot{\Psi} \quad (16)$$

and

$$\delta\mathcal{P}_{IDM} = -9\gamma H\dot{\Psi} - 3\gamma\ddot{\Psi} - 3\dot{\gamma}\dot{\Psi} , \quad (17)$$

respectively.

3.2 Before shift-symmetry breaking

We first discuss the case of the zeroth Noether charge. Let us show that perturbations of IDM are exactly adiabatic in that case. One obtains from Eq. (16),

$$\delta\rho_{IDM} = 9\gamma H\dot{\Psi} . \quad (18)$$

The analogous expression for the energy density perturbation of the external matter can be inferred from the 00-th component of Einstein's equations, which reads in the synchronous gauge

$$3H\dot{\Psi} = \frac{1}{2}\delta\rho_{ext} + \frac{1}{2}\delta\rho_{IDM} . \quad (19)$$

Combining this with Eq. (18), one obtains

$$\delta\rho_{IDM} = \frac{3\gamma}{2-3\gamma}\delta\rho_{ext} .$$

We plug the latter into the definition of the curvature perturbation of IDM,

$$\zeta_{IDM} = \Psi + \frac{\delta\rho_{IDM}}{3(\rho_{IDM} + \mathcal{P}_{IDM})} , \quad (20)$$

and then make use of the relation

$$\rho_{IDM} + \mathcal{P}_{IDM} = \frac{3\gamma}{2-3\gamma}(\rho_{ext} + \mathcal{P}_{ext}) .$$

This expression follows from Eqs. (10) and (11), where one should set the Noether charge density to zero and the parameter γ to a constant. We obtain,

$$\zeta_{IDM} = \Psi + \frac{\delta\rho_{ext}}{3(\rho_{ext} + \mathcal{P}_{ext})} \equiv \zeta_{ext} , \quad (21)$$

where ζ_{ext} is the curvature perturbation of the external matter. Finally, we use the relation between the partial curvature perturbations corresponding to the external matter fields and the quantity ζ_{ext} ,

$$\zeta_{ext} = \sum_{i \neq IDM} \frac{\dot{\rho}_i}{\dot{\rho}_{ext}} \zeta_i .$$

Here the index i stands for the particular matter field, i.e., photons, neutrinos, baryons etc., and the subscript $i \neq IDM$ means that the contribution of IDM has been omitted. Under an assumption that there is no admixture of the baryon or neutrino isocurvature modes, all the partial curvature perturbations $\zeta_{i \neq IDM}$ are equal between each other. Since an equality $\sum_{i \neq IDM} \dot{\rho}_i / \dot{\rho}_{ext} = 1$, one has $\zeta_{i \neq IDM} = \zeta_{ext}$. Comparing this with Eq. (21), we get

$$\zeta_{IDM} = \zeta_{i \neq IDM} . \quad (22)$$

That is, the IDM curvature perturbation equals to the curvature perturbations of the standard matter fields. This means that IDM perturbations are exactly adiabatic in the shift-symmetric case/before the shift-symmetry breaking takes place. That situation is quite similar to what happens in the standard cosmology with the particle DM: at very early times all the particle species behave as the single fluid, and initial scalar perturbations are adiabatic by definition.

3.3 After shift-symmetry breaking

While IDM perturbations are exactly adiabatic before shift-symmetry breaking, it is not immediately clear that they remain so at later times. Indeed, starting from initial conditions (22), they can change due to the presence of the non-adiabatic pressure,

$$\mathcal{P}_{IDM}^{non-ad} \equiv \delta\mathcal{P}_{IDM} - \frac{\dot{\mathcal{P}}_{IDM}}{\dot{\rho}_{IDM}} \delta\rho_{IDM} , \quad (23)$$

which is not manifestly zero. This is one distinction of the IDM fluid from the familiar fluids, i.e., radiation and dust. Consequently, the IDM curvature perturbation evolves behind the horizon accordingly to the equation [24],

$$\dot{\zeta}_{IDM} = -\frac{H}{\rho_{IDM} + \mathcal{P}_{IDM}} \mathcal{P}_{IDM}^{non-ad} , \quad (24)$$

while those of the standard matter fields remain constant. This results into the violation of the condition (22) leading to the appearance of an isocurvature mode and, consequently, to a potential conflict with the Planck data. The non-adiabatic pressure, however, is negligible in two regimes: at the times $t < t_*$ and $t \gg t_*$. The former follows from an exact adiabaticity of IDM perturbations at the early times implied by Eq. (22)⁸. The latter is also clear, since IDM relaxes to the standard dust at sufficiently late times. Hence, the non-adiabatic pressure is relevant only in the intermediate regime $t \simeq t_*$, when the effects due to the shift-symmetry breaking may become strong enough. They are encoded in the appearing Noether charge density in Eqs. (8) and (16) and explicitly in terms involving the derivative of the function γ in Eqs. (9) and (17). These new terms source the non-adiabatic pressure. To summarize, an expected change induced in the curvature perturbation ζ_{IDM} is measured in terms of the quantities calculated at the times $t \simeq t_*$. Shortly, we will confirm this observation by making an exact calculation. We will also see that the variation of the curvature perturbation ζ_{IDM} is small, as it relies on the derivative of the potential Ψ .

To study the evolution of the curvature perturbation, Eq. (24) is not very convenient. For this purpose, it is simpler to exploit Eq. (6). Integrating the latter and using Eq. (13), one obtains at the times $t > t_*$,

$$\delta n = 3n (\Psi(t_*) - \Psi) + 2n \cdot \frac{\dot{\Psi}(t_*)}{H(t_*)} . \quad (25)$$

⁸Not referring to Eq. (22), one can show that the non-adiabatic pressure of IDM at the times $t < t_*$ is proportional to that of the external matter, i.e., $\mathcal{P}_{IDM}^{non-ad} \propto \mathcal{P}_{ext}^{non-ad}$, where the subscript 'ext' stands for the combination of photons, neutrinos, baryons etc. As perturbations of the external matter fields are assumed to be adiabatic, one has $\mathcal{P}_{ext}^{non-ad} = 0$. Consequently, $\mathcal{P}_{IDM}^{non-ad} = 0$, as it should be.

Using then Eqs. (8), (9), (13), (16), (19) and (25), we derive the expression for the curvature perturbation of IDM,

$$\zeta_{IDM} = \Psi + \frac{3n(\Psi(t_*) - \Psi) + 6n \cdot \frac{\dot{\Psi}(t_*)}{H(t_*)} + 9\gamma H \dot{\Psi}}{3[n - 3\gamma \dot{H}]} . \quad (26)$$

Hereafter, γ denotes the value of the parameter at the end of the transition $\gamma_1 \rightarrow \gamma_2$, i.e., $\gamma \equiv \gamma_2$. Note that at the times $t < t_*$, when the Noether charge density equals to zero, Eq. (26) reduces to the expression for the curvature perturbation of the total matter ζ_{tot} , as it should be. Now, we are interested in the different regime, when IDM mimics the behaviour of the dust particles, i.e., strong inequalities $n \gg \gamma |\dot{H}| \sim \gamma H^2 \sim \gamma \rho_{tot}$ are obeyed. For the relevant values of the parameter γ this still happens deeply in the RD stage. In that limit, we get for the curvature perturbation of IDM,

$$\zeta_{IDM} = \Psi(t_*) + 2 \frac{\dot{\Psi}(t_*)}{H(t_*)} . \quad (27)$$

The latter is constant as expected: the non-adiabatic pressure is negligible in the late-time regime. Now, let us show that the second term on the r.h.s. of Eq. (27) equals to zero, i.e., the derivative of the gravitational potential vanishes at $t = t_*$. This we will do in the reasonable approximation, when all the external matter is in the state of radiation⁹. We will need the ij -component of Einstein's equations, which reads in the synchronous gauge,

$$\ddot{\Psi} + 3H\dot{\Psi} = -\frac{1}{2}(\delta\mathcal{P}_{ext} + \delta\mathcal{P}_{IDM}) . \quad (28)$$

For the relativistic external matter, one has $\delta\mathcal{P}_{ext} = \frac{1}{3}\delta\rho_{ext}$. Using this, we combine Eqs. (19) and (28) to exclude the quantities describing the external matter,

$$\ddot{\Psi} + 4H\dot{\Psi} = \frac{1}{6}\delta\rho_{IDM} - \frac{1}{2}\delta\mathcal{P}_{IDM} .$$

We substitute expressions (16) and (17) for the IDM energy density and pressure perturbations and rewrite the equation above as follows,

$$\frac{d}{dt} \left[\left(1 - \frac{3\gamma}{2} \right) \dot{\Psi} \right] + 4H \left(1 - \frac{3\gamma}{2} \right) \dot{\Psi} = \frac{\delta n}{6} .$$

Integrating this out, we get

$$\left(1 - \frac{3\gamma}{2} \right) \dot{\Psi} a^4 = \frac{1}{6} \int_{t_i}^t a^4(\tilde{t}) \delta n(\tilde{t}) d\tilde{t} + C . \quad (29)$$

⁹In particular, this statement is exact for the times t_* corresponding to the temperatures $T \gtrsim 100$ GeV, when all the Standard Model degrees of freedom are relativistic.

Here $t_i < t$ and C are the arbitrary constants. Recall now that the Noether charge density is zero at the times $t < t_*$. As we are interested in the behaviour of the potential Ψ at $t = t_*$ and since the quantity n is always finite, the first term on the r.h.s. of Eq. (29) vanishes. Hence, the solution for the derivative of the gravitational potential reduces to

$$\dot{\Psi}(t_*) = \frac{C}{\left(1 - \frac{3\gamma(t_*)}{2}\right) a^4(t_*)} . \quad (30)$$

For the arbitrary value of the constant C , this solution is discontinuous, as it follows from the behaviour of the parameter γ at $t = t_*$. Note, however, that the r.h.s. of Eq. (30) translates into the decaying mode of the potential Ψ , which is commonly dropped in cosmology¹⁰. That is, we should set the constant C to zero. In this situation, the derivative of the potential Ψ vanishes, i.e., $\dot{\Psi}(t_*) = 0$, and the expression for the IDM curvature perturbation (27) simplifies to $\zeta_{IDM} = \Psi(t_*)$.

Consequently, the potential Ψ is a continuous constant function at the time $t = t_*$. From Eq. (25), we then conclude that $\delta n(t_*) = 0$. The same is true for the IDM and external matter energy density perturbations, i.e., $\delta\rho_{IDM}(t_*) = 0$ and $\delta\rho_{ext}(t_*) = 0$. These follow from Eqs. (16) and (19), respectively. In the situation, when all the degrees of freedom are relativistic and there is no admixture of the isocurvature mode in the particle sector, one has an equality $\delta_{ext} = \delta_{ph}$, where $\delta_{ext} \equiv \frac{\delta\rho_{ext}}{\rho_{ext}}$ and $\delta_{ph} \equiv \frac{\delta\rho_{ph}}{\rho_{ph}}$; the subscript "ph" stands for the photons. Hence, $\delta_{ph}(t_*) = 0$. Recall now the expression for the curvature perturbation of the photons,

$$\zeta_{ph} = \Psi + \frac{1}{4}\delta_{ph} .$$

The latter stays constant behind the horizon, i.e., $\zeta_{ph}(t) = \zeta_{ph}(t_*)$. Thus, $\zeta_{ph} = \Psi(t_*)$. To summarize, there is no admixture of the IDM isocurvature mode, i.e.,

$$S_{IDM,ph} \equiv 3(\zeta_{IDM} - \zeta_{ph}) = 0 . \quad (31)$$

Perturbations of IDM remain adiabatic after the shift-symmetry breaking, provided only that we can neglect the non-relativistic degrees of freedom during the transition $\gamma_1 \rightarrow \gamma_2$.

The result (31) can be also understood from a slightly different prospective. As we noted earlier, the IDM curvature perturbation starts from exact adiabatic initial conditions (22) and may change only due to the non-zero adiabatic pressure (23). The latter can be relevant only at the times $t \simeq t_*$. Next, we observe that both the energy density perturbation (the numerator in Eq. (26)) and the pressure perturbation (17), rely only on the variation of the potential Ψ . Accordingly to Eq. (23), so does the non-adiabatic pressure and, consequently,

¹⁰Recall that the scale factor during the RD stage grows as $a \propto \sqrt{t}$. Hence, the decaying mode of the potential Ψ drops as $\Psi_{dec} \propto \frac{C}{t}$.

the isocurvature perturbation. Hence, the resultant perturbations must be adiabatic, once the potential Ψ is constant at the relevant times.

Finally, let us comment on the generality of the results obtained. First, we notice that the assumption of the instantaneous transition $\gamma_1 \rightarrow \gamma_2$ is not a strong one at all. Our results hold for the different choices of the function $\gamma(\varphi)$ given that the gravitational potential Ψ remains constant during the phase of the shift-symmetry breaking. That condition is satisfied with a high accuracy provided that the transition occurs well within the RD stage.

Second, instead of the HD term as in Eq. (1), one could consider another one,

$$+ \frac{\tilde{\gamma}(\varphi)}{2} \nabla_\mu \nabla_\nu \varphi \nabla^\mu \nabla^\nu \varphi . \quad (32)$$

We provide the associated analysis in Appendix B. In particular, we show that all the three statements take place: i) in the shift-symmetric case, the equation of state of IDM is that of the total matter; 2) perturbations of IDM are exactly adiabatic in this situation; 3) an approximate adiabaticity holds after the short phase of shift symmetry breaking.

To conclude, initial conditions for IDM are the same as in the CDM case. Hence, CDM and IDM result with the same predictions regarding the cosmological observations. An important difference, however, may arise at the galaxy scales. This is due to the fact that IDM possesses non-zero sound speed. Furthermore, IDM and CDM exhibit an apparently different behaviour in the non-linear regime [2]. We leave this and other interesting questions for the future.

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Appendix A

In this Appendix, we switch to the Newton's gauge in order to cross-check the results obtained in the main body of the paper. In that case, the perturbation of the Noether charge density is given by

$$\delta n = \delta \lambda + 6\gamma \dot{H} \Phi + 3\gamma H \dot{\Phi} - 3\gamma \ddot{\Psi} ,$$

where the shift-symmetry is assumed. The expression for the energy density perturbation of IDM reads in the Newton's gauge,

$$\delta \rho_{IDM} = \delta n - 9\gamma H^2 \Phi + 9\gamma H \dot{\Psi} . \quad (33)$$

Recall that the Noether charge density equals to zero before the shift-symmetry breaking takes place. Integrating Eq. (6) with the initial condition $n = 0$ set at the times $t < t_*$, one

obtains at the times $t > t_*$,

$$\delta n = -2n \cdot \Phi(t_*) + 3n \cdot (\Psi(t_*) - \Psi) + \frac{n\delta\varphi(t_*)}{H(t_*)} \cdot \mathcal{P}_{tot}(t_*) + 2n \cdot \frac{\dot{\Psi}(t_*)}{H(t_*)}, \quad (34)$$

where $n \cdot a^3 = \text{const}$ is defined by Eq. (13). Substituting Eq. (34) into Eq. (33), and then the latter into (20), we obtain for the IDM curvature perturbation,

$$\zeta_{IDM} = \Psi + \frac{1}{3[n - 3\gamma\dot{H}]} \times \left[3n(\Psi(t_*) - \Psi) + n \cdot \delta_{tot}(t_*) + \frac{3}{2}\gamma \cdot \delta\rho_{tot} + \frac{n \cdot \delta\varphi(t_*)}{H(t_*)} \cdot \mathcal{P}_{tot}(t_*) \right], \quad (35)$$

where $\delta_{tot} \equiv \delta\rho_{tot}/\rho_{tot}$. Here we made use of the 00th component of Einstein's equations, which reads in the Newton's gauge (in the super-horizon regime),

$$3H\dot{\Psi} - 3H^2\Phi = \frac{1}{2}\delta\rho_{tot}.$$

The curvature perturbation (35) corresponds to exactly adiabatic initial conditions for IDM before the Noether charge is produced, as expected. Well after the shift-symmetry gets broken, i.e., in the regime $n \gg \gamma H^2$, one has

$$\zeta_{IDM} = \Psi(t_*) + \frac{1}{3}\delta_{tot}(t_*) + \frac{1}{3} \frac{\delta\varphi(t_*)}{H(t_*)} \cdot \mathcal{P}_{tot}(t_*).$$

This expression is equal to the curvature perturbation (27) calculated in the synchronous gauge, as it should be. In particular, neglecting the super-horizon variation of the potential Ψ at the time t_* and using $\delta\varphi \approx \Phi t$ and $H \approx \frac{1}{2t}$, we get

$$\zeta_{IDM} \approx \Psi(t_*) + \frac{\delta\rho_{tot}(t_*)}{4\rho_{tot}(t_*)} \approx \zeta_{ph}.$$

This corresponds to the adiabatic initial conditions for IDM.

Appendix B

In this Appendix we consider another possible HD term given by Eq. (32). The associated energy momentum tensor is given by [15]

$$T_{\mu\nu} = \lambda\partial_\mu\varphi\partial_\nu\varphi + \tilde{\gamma}\nabla^\lambda(\nabla_\lambda\varphi\nabla_\mu\nabla_\nu\varphi) - \tilde{\gamma}\square\nabla_\mu\varphi\nabla_\nu\varphi - \tilde{\gamma}\square\nabla_\nu\varphi\nabla_\mu\varphi - \frac{1}{2}\tilde{\gamma}g_{\mu\nu}\nabla^\alpha\nabla^\beta\varphi\nabla_\alpha\nabla_\beta\varphi.$$

At this level, we assumed the shift-symmetry, as it is going to be enough for our purposes.

First, it is straightforward to show that in an exactly shift-symmetric case, IDM still tracks the dominant matter of the Universe. Indeed, from the energy-momentum tensor one deduces for the energy density and the pressure,

$$\rho_{IDM} = n + \frac{\tilde{\gamma}}{2}\rho_{tot} \quad (36)$$

and

$$\mathcal{P}_{IDM} = \frac{\tilde{\gamma}}{2} \mathcal{P}_{tot} . \quad (37)$$

Here n is the Noether charge density now given by the expression

$$n \equiv J^0 = \lambda \partial^0 \varphi - \nabla^\nu (\tilde{\gamma} \nabla_\nu \nabla^0 \varphi)$$

As it follows, in the situation, when the Noether charge is zero, the equation of state of IDM is that of the total matter in the Universe. Not surprisingly thus, super-horizon modes of IDM are exactly of the adiabatic type.

To show this explicitly, we set to zero the Noether charge density n as well as its super-horizon perturbation,

$$\delta n = \delta \lambda + 6\tilde{\gamma} H \dot{\Psi} .$$

The relation between the IDM energy density perturbation and the quantity δn is given by

$$\delta \rho_{IDM} = \delta n + 3\tilde{\gamma} H \dot{\Psi} . \quad (38)$$

Hence,

$$\delta \rho_{IDM} = 3\tilde{\gamma} H \dot{\Psi} .$$

Substituting this into the definition (20), using Eqs. (36) and (37), one obtains $\zeta_{IDM} = \zeta_{tot}$. The latter implies an exact adiabaticity of IDM perturbations in the situation, when the Noether charge density equals to zero.

To obtain the expression for the perturbation ζ_{IDM} in the generic case, one exploits an equation,

$$\frac{d}{dt} (\sqrt{-g} \cdot n) = \frac{\tilde{\gamma}_\varphi}{2} \sqrt{-g} \nabla_\mu \nabla_\nu \varphi \nabla^\mu \nabla^\nu \varphi . \quad (39)$$

Namely, we promote the constant $\tilde{\gamma}$ to the function of the field φ . We again assume with no loss of generality, the instantaneous transition of the parameter $\tilde{\gamma}$ as in Eq. (12). Integrating Eq. (39) over the time, we get

$$n \cdot a^3 = \frac{3}{2} \Delta \tilde{\gamma} H^2 a^3 \big|_{t=t_*} \quad t > t_* .$$

The analogous equation at the level of super-horizon fluctuations coincides with Eq. (25). Substituting Eq. (25) into Eq. (38), we obtain for the IDM energy density perturbation

$$\delta \rho_{IDM} = 3n \cdot (\Psi(t_*) - \Psi) + n \cdot \delta_{tot}(t_*) + \frac{1}{2} \tilde{\gamma} \cdot \delta \rho_{tot} . \quad (40)$$

We substitute this into Eq. (20). At sufficiently late times, i.e., when the strong inequality $n \gg \tilde{\gamma} \rho_{tot}$ is obeyed, we arrive at the expression (27). Again neglecting the super-horizon variation of the potential Ψ and then following the same steps as in the bulk of the paper, one concludes with adiabaticity of IDM perturbations.

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